

# ION TRAPPING, CLEARING, BEAM–ION INTERACTIONS

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## Abstract

The present lecture is based on a revised version of the one given 6 years ago at the CERN Accelerator School [1]. After a brief review of the adverse effect of neutralisation in particle storage rings, the basic topics of ion production by ionisation of the residual gas are recalled: ion production rate, natural clearing rates, ion kinematics and conditions of trapping for bunched and unbunched particle beams with positive or negative space charge. Different methods of clearing are described and their performance discussed, namely, DC clearing electrodes, empty buckets (in electron storage rings) and beam shaking. Examples of neutralisation effects and diagnostics are taken from CERN machines, such as ion clearing current measurements on electrodes, tune shifts and emittance measurements.

## INTRODUCTION

In accelerators and storage rings, ions created by the circulating particles from neutral molecules of the residual gas may be trapped in the beam-space-charge potential and may generate all sorts of ill effects: reduced beam lifetime (increased pressure), emittance growth and losses through excitation of resonances, and coherent beam instabilities. While they can occur in proton beams (e.g. CERN ISR trapping electrons), these neutralisation phenomena mainly affect machines with negative beam space charge such as electron storage rings and antiproton accumulators.

Low energy machines are more subject to ion trapping because of their small size, which leaves little space between bunches for ions to escape the beam potential, and suffer most because of their inherent high sensitivity to space-charge effects. The ion-induced space-charge tune shifts  $\Delta Q$  can be unacceptably large if  $\gamma$  is small (low energy) and/or the neutralisation  $\eta$  is high.

### 1. NEUTRALISATION OF A BEAM: A SIMPLE DESCRIPTION

The circulating particles in a stored beam collide with residual gas molecules producing positive ions and electrons. A negatively charged beam (e.g., electrons or antiprotons) captures the ions and repels the electrons towards the vacuum chamber walls<sup>1</sup>. If other possible natural or artificial clearing mechanisms are not present, the neutralising ions accumulate up to the point where the remaining trapping potential is effectively zero, i.e., until the number of static neutralising particles is equal to the number of beam particles. The beam is then fully neutralised. The average neutralisation factor is defined by

$$\eta = \frac{n_i}{n_e}, \quad (1)$$

where  $n_i$  is the total neutralising charge measured in units of the electronic charge and  $n_e$  is the number of stored beam particles. The neutralisation is often not homogeneous along the machine azimuth  $s$ , and we define a local neutralisation factor by

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<sup>1</sup>Positively charged coasting beams trap electrons, and this effect has been extensively studied in the CERN ISR (see references). Bunched proton or positron beams do not suffer from neutralisation problems because the electrons are not stably trapped (cf section 5).

$$\eta(s) = \frac{2\pi R}{n_e} \frac{dn_i}{ds}, \quad (2)$$

where  $2\pi R$  is the machine circumference and  $(dn_i/ds)$  is a local linear neutralising charge density (measured in units of electronic charge per meter)<sup>2</sup>.

In order to get a feeling for the orders of magnitude involved in neutralisation problems, we consider a set of machine parameters corresponding to typical values for the CERN electron-positron accumulator (EPA). We disregard, for the time being, the fact that the EPA electron beam is bunched and only calculate longitudinally averaged values. Also, for the sake of simplicity, we assume a round beam with a homogeneous transverse charge distribution. The beam current is  $I = 100$  mA, the energy is  $E = 500$  MeV and the beam radius is 0.5 mm. The corresponding linear particle density is

$$\lambda = \frac{dn_e}{ds} = \frac{I}{e\beta c} = 2 \times 10^9 \text{ particles/m.} \quad (3)$$

The electric field at the beam edge is obtained via Gauss's law.

$$\varepsilon = \frac{e\lambda}{2\pi\varepsilon_0 a} = 1.2 \times 10^4 \text{ volt/m.} \quad (4)$$

The magnetic field at the edge is obtained via Ampere's law:

$$B = \frac{I\mu_0}{2\pi a} = 4 \times 10^{-5} \text{ T.} \quad (5)$$

The total direct space-charge force on a circulating electron is

$$\vec{F} = e(\vec{\varepsilon} + \vec{v} \times \vec{B}) = \vec{F}_e + \vec{F}_m = \vec{F}_e \frac{1}{\gamma^2}, \quad (6)$$

where  $\gamma$  is the total relativistic beam energy in units of the rest energy and  $\vec{F}_e$  is the electrostatic force. The fact that the two forces counteract results in the so-called relativistic cancellation. With neutralisation, the electrostatic force is changed from  $\vec{F}_e$  to  $\vec{F}_e(1-\eta)$  and the magnetic force is unchanged. Then

$$F = F_e \left( \frac{1}{\gamma^2} - \eta \right). \quad (7)$$

The force which we have calculated at the edge of the beam is in fact proportional to the distance to the centre of the beam. The corresponding local quadrupole has a strength (with the Courant and Snyder definition [2])

$$k(s) = -\frac{1}{E} \frac{dF}{dr}. \quad (8)$$

We simplify the formulae by introducing the classical electron radius

$$r_e = \frac{e^2}{4\pi\varepsilon_0 m_e c^2} = 2.82 \times 10^{-15} \text{ m.} \quad (9)$$

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<sup>2</sup> Some authors define an average neutralisation factor for bunched beams as the ratio of the average neutralising charge density to the bunch charge density. This is related to our definition through the bunching factor  $B = (N_b L_b)/(2\pi R)$  where  $N_b$  is the number of bunches and  $L_b$  is the bunch length. This definition is useful when one compares the incoherent space-charge tune shift with the neutralisation induced tune shift.

and the local beam volume density

$$d_e = \frac{dn_e}{ds} \frac{1}{\pi a^2} = 2.7 \times 10^5 \text{ particles/m}^3 \quad (10)$$

so that, with Eqs. (4), (7), (8), (9), and (10),

$$k(s) = -\frac{2\pi}{\gamma} r_e d_e \left( \frac{1}{\gamma^2} - \eta \right) \quad (11)$$

and the corresponding tune shift is

$$\Delta Q = \frac{1}{4\pi} \int \beta(s) k(s) ds, \quad (12)$$

where  $\beta(s)$  is the usual Twiss parameter and the integration is done along the machine azimuth. Instead of calculating the integral, we use average values for all quantities involved<sup>3</sup>:

$$\Delta Q = \frac{r_e (2\pi R) \bar{\beta}}{2\gamma} d_e \eta. \quad (13)$$

For  $\bar{\beta} = 4 \text{ m}$  and  $2\pi R = 126 \text{ m}$ , we find

$$\Delta Q \approx 2\eta \quad (14)$$

and machine performance will be limited for values of  $\eta$  above a few percent.

## 2. THE IONISATION PROCESS

The circulating beam interacts with the electrons of the molecules of the residual gas and with the ions trapped in the beam. In turn the trapped ions interact with the molecules in many different ways. In the following sections, we briefly review these phenomena.

### 2.1 Transfer of energy to free electrons

An estimate can be obtained through the calculation of the electrostatic interaction between a free electron and the primary particle (Jackson [3] section 13.1). The energy transfer  $E'(b)$  is a function of the impact parameter  $b$  (Fig. 1).

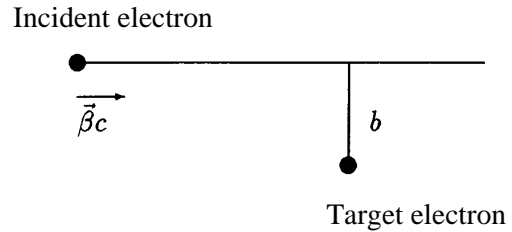


Fig. 1 The impact parameter  $b$ .

Equation 13.2 of Jackson, where the field at the electron is obtained by a Lorentz transformation, can be rewritten in mks units and with the classical electron radius:

$$E'(b) = \frac{2m_e c^2 r_e^2}{\beta^2 b^2} \quad (15)$$

or

$$b^2 = r_e^2 \frac{1}{E'} \frac{2m_e c^2}{\beta^2}, \quad (16)$$

<sup>3</sup> We now disregard the small direct space-charge contribution.

where  $\beta c$  is the velocity of the primary particle and  $E'(b)$  is the transferred energy.

The cross-section  $d\sigma$  for energy transfer between  $E'$  and  $E'+dE'$  is

$$d\sigma = 2\pi b db \quad (17)$$

or

$$d\sigma = 2\pi \frac{m_e c^2}{\beta^2} r_e^2 \frac{dE'}{E'^2}. \quad (18)$$

One sees immediately an unphysical situation for  $E' = 0$  ( $b$  large, distant collisions) and  $E' = \infty$  ( $b$  small, close collisions). The difficulty is solved by the definition of a minimum energy  $E'_{\min}$  and a maximum energy  $E'_{\max}$ . The maximum energy  $E'_{\max}$  can be obtained from pure kinematic considerations,<sup>4</sup> whereas the minimum energy requires a detailed analysis of the medium in which the interaction takes place.

The above formulae cannot be used to compute the exact ionisation cross-section, as we shall see below, but they give a fair description of the phenomenon. Detailed measurements of  $d\sigma/dE$  have been made [95] and are shown in Fig. 2.

An ionisation event takes place only if the energy transferred is above the ionisation potential. Most free electrons created in such an event will be left with a rather small energy and will therefore be trapped by the beam (Fig. 2). In Ref. [96], it has been calculated that about 80% of the electrons have an energy below 45eV, the average being around a few eV. The ion energy will be smaller in the ratio of the masses and therefore negligible. These electron energies, however, should be used to calculate their drift velocities (cf. section 3) because they are several orders of magnitude higher than the thermal energy ( $\sim 10^{-2}$  eV at 300 K). This is not the case for ions. The proportion of ions not trapped because they are created with an energy larger than the potential well is negligible (less than 4% for a potential well of a few hundred volts [96]).

## 2.2 Ionisation cross-section

The ionisation cross-section depends on the molecule of the residual gas and on the velocity of the ionising particle but neither on its charge nor on its mass<sup>5</sup>. Measurements have been made [98] for various incident electron energies and the results were fitted to the theoretical expression by Bethe (see [4], p.45):

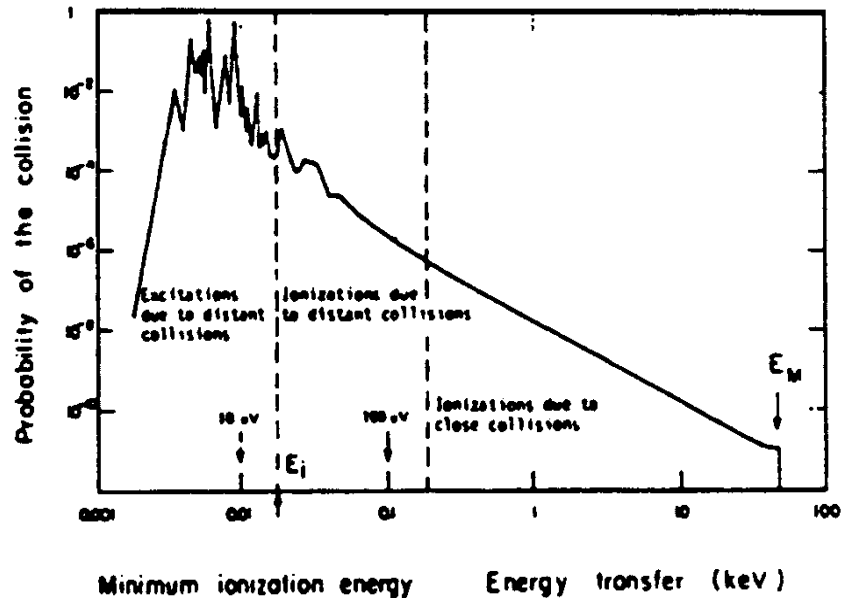


Fig. 2 Relative probability of different processes induced by fast (100 keV) electrons in water, as a function of the energy transfer in a collision [95]. The maximum kinematically allowed energy transfer,  $E_{\max} = 50$  keV in this case, is also shown.

<sup>4</sup> For relativistic incident particles, quantum or relativistic effects further reduce the maximum energy transfer [4].

<sup>5</sup> Here we only consider ionisation by charged particle impact. Photo-ionisation in electron machines is analysed in ref. [97].

$$\sigma^i = 4\pi \left( \frac{\hbar}{m_e c} \right)^2 \left\{ M^2 \left[ \frac{1}{\beta_e^2} \ln \left( \frac{\beta_e^2}{1 - \beta_e^2} \right) - 1 \right] + \frac{C}{\beta_e^2} \right\} \quad (19)$$

$$4\pi \left( \frac{\hbar}{m_e c} \right)^2 = 1.87 \times 10^{-24} \text{ m}^2 . \quad (20)$$

The experimentally determined coefficients  $C$  and  $M^2$  for different molecules are shown in Table 1. Figure 3 shows a plot of the cross-sections given by the formulae above.

Table 1  
Value of the  $M^2$  and  $C$  constants for calculation  
of ionisation cross-sections

| Molecule                      | $M^2$ | $C$   | $Z$ | $A$ |
|-------------------------------|-------|-------|-----|-----|
| H <sub>2</sub>                | 0.5   | 8.1   | 2   | 2   |
| N <sub>2</sub>                | 3.7   | 34.8  | 14  | 28  |
| CO                            | 3.7   | 35.1  | 14  | 28  |
| O <sub>2</sub>                | 4.2   | 38.8  | 16  | 32  |
| H <sub>2</sub> O              | 3.2   | 32.3  | 10  | 18  |
| CO <sub>2</sub>               | 5.75  | 55.9  | 22  | 44  |
| C <sub>4</sub> H <sub>4</sub> | 17.5  | 162.4 | 46  | 76  |

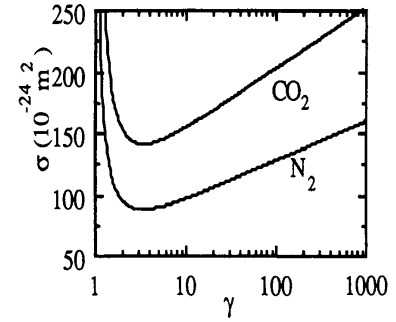


Fig. 3 Ionisation cross-section vs  $\gamma$  for CO<sub>2</sub> and N<sub>2</sub>

### 2.3 Ionisation rate

The time it takes for one circulating particle to create one ion is given by:

$$\tau_m = \frac{1}{d_m \sigma_m \beta c} \quad (21)$$

where  $d_m$  is the molecular density (m<sup>-3</sup>);  $\tau_m$  is the ionisation cross-section for molecule  $m$  (m<sup>2</sup>);  $\beta c$  is the velocity of the circulating beam (m.s<sup>-1</sup>)

The molecular density  $d_m$  is related to the partial pressure  $P_m$  in torr by the relation (valid at 20°C).

$$d_m = 3.3 \times 10^{22} P_m \quad (22)$$

If there are several types of molecules in the residual gas then the total ionisation time  $\tau_i$  is given by the relation.

$$\frac{1}{\tau_i} = \sum_m \frac{1}{\tau_m} .$$

In electron storage rings with typical residual pressures of 1 nTorr composed of CO and H<sub>2</sub>, the ionisation time is of the order of one second or less.

### 2.4 Beam heating

Distant collisions with large impact parameter — much more probable than close ones leading to ionisation — are important, since they feed energy differentially to ions. In some circumstances (neutralisation pockets) this may be a clearing mechanism, i.e. when the trapped species get enough energy to escape the beam potential  $V$ :

$$R_c = \frac{1}{eV} \frac{dE}{dt} = \frac{1}{eV} \int_{b_{\min}}^{b_{\max}} \Delta E(b) (2\pi b \beta c N_p db) .$$

This represents the "natural" clearing rate for a singly-charged species. The expression between brackets is the number of projectiles passing at distance  $b$  to the ion target during time  $dt$ .  $N_p$  is the projectile density of charge  $Z = 1$ .

To good approximation  $b_{\max}$  and  $b_{\min}$  can be chosen to have the same values as the ion and nucleus radii respectively, leading to [3]:

$$R_c = \frac{2\pi m_0 c^3 r_0^2}{\beta} \cdot \frac{N_p Z'}{eV} \cdot \ln(3 \cdot 10^4 \cdot Z'^{-2/3}) \quad (23)$$

with  $m_0$  and  $r_0$  being  $m_e$ ,  $r_e$  if the trapped species is an electron,  $m_p$ ,  $r_p$  for a proton;  $2 m_p$ ,  $r_p$  for an ion of charge  $Z'$ .

As an example, typical clearing times for the EPA machine with  $6 \cdot 10^{11}$  electrons (300 mA), 1 mm beam radius, giving a beam potential of  $\sim 50$  V, are shown in Table 2.

Table 2  
EPA natural ion clearing rates

| Clearing rate $R_c$<br>( $s^{-1}$ ) |                   | Clearing time $\zeta_c$<br>(s) |
|-------------------------------------|-------------------|--------------------------------|
| H <sup>+</sup>                      | $3 \cdot 10^{-3}$ | 350                            |
| H <sub>2</sub> <sup>+</sup>         | $6 \cdot 10^{-3}$ | 166                            |
| CO <sup>+</sup>                     | 0.04              | 25                             |
| CO <sub>2</sub> <sup>+</sup>        | 0.07              | 15                             |

The process is thus slow compared to typical ionisation rates. But it may be important to explain why in some circumstances, e.g. pockets with very low gas pressure ( $5 \cdot 10^{-11}$  Torr, see AA case in reference [116]) fully ionised light ions can accumulate to a dangerous level, as they replace heavy ones which are chased away by beam heating.

## 2.5 Gas cooling

Seldom taken into account, gas cooling could be an important process for high pressures and long ion resting times. Charge-exchange phenomena by which a positive ion captures an electron from a gas molecule may occur at ion energies of only a few eV. The new ion is created with the primary molecule's energy, while the newly created neutral species carries away the initial ion energy. Resonant capture cross-sections between an ion and its own neutral molecule can be very high at low energy:

$$\sigma = 1.2 \times 10^{-15} \text{ cm}^2 \text{ for He}^+ \text{ in He}$$

for an ion energy of 3 eV. With this cross-section, it would take only 27 seconds at  $10^{-8}$  torr for an ion to capture an electron and thus leave a cool ion behind. Capture cross-sections are even higher for heavy gas species.

## 2.6 Limits on ion accumulation

In the vast majority of cases (electron storage rings with typical pressures of  $10^{-9}$  torr and ionisation times of a second or less), ionisation is the dominant effect in the absence of any clearing mechanism. The neutralisation reached at equilibrium is strongly dependent on the quality of the vacuum (gas species and densities). To illustrate this, we consider the over-simplistic case of constant ionisation cross-sections. The production rates are, for singly-ionised species (density  $N_i^+$ ):

$$\frac{d(N_i^+)}{dt} = N_m N_p \sigma_i c - N_i^+ N_p \sigma_i c = N_p \sigma_i c (N_m - N_i^+)$$

doubly ionised:

$$\frac{d(N_i^{++})}{dt} = N_i^+ N_p \sigma_i c - N_i^{++} N_p \sigma_i c = N_p \sigma_i c (N_i^+ - N_i^{++})$$

etc. until ( $Z'$  being the total number of electrons of the gas atom):

$$\frac{d(N_i^{Z'+})}{dt} = N_i^{(Z'-1)+} N_p \sigma_i c.$$

In the steady state:

$$N_i^+ = N_i^{++} = \dots = N_i^{(Z'-1)+} \leq N_m.$$

Therefore partially ionised ions can, at most, reach the molecular density  $N_m$ . Only the fully ionised state  $N_i^{Z'+}$  could get close to the particle density (usually much larger than the gas density) divided by the final charge state:  $N_i^{Z'+} \leq N_p/Z'$  corresponding to full neutralisation of the particle beam. ("Over" neutralisation can for most cases be excluded, since the resulting potential would chase the species to the chamber wall.)

The degree of neutralisation of a particular beam can be estimated from the incoherent tune shift (see Sec. 9). Almost full neutralisation has been measured in the CERN AA when all the clearing electrodes are turned off (Fig. 4).

### 3. THE ION OR ELECTRON MOTION

The temperature of the molecules of the residual gas will be slightly increased by the interactions with the beam. However, the reservoir of molecules is so big that the energy in the gas will be the energy related to the temperature of the vacuum chamber walls, usually 300 K. The energy of the electrons acquired through momentum transfer from the circulating beam in the ionisation process will be of a few eV and the energy of the ions lower by the mass ratio

$$\frac{m_e}{Am_p} \quad (24)$$

where  $Am_p$  is the mass of the ion, and  $m_e$ , the mass of the electron. The electrons and the ions created inside the beam will either be chased out or oscillate in its potential well. Their motion will be influenced by magnetic fields. The analysis of these different energies and motions is the object of this section.

#### 3.1 Energy, temperature, velocity

The distribution of velocity of molecules of mass  $m$  in a gas of density  $d_m$  at temperature  $T$  is given by the Boltzmann equation

$$dn = \frac{1}{2} d_m \sqrt{\frac{m^3}{2\pi^3 k^3 T^3}} e^{-\frac{m}{2kT}(v_x^2 + v_y^2 + v_z^2)} dv_x dv_y dv_z, \quad (25)$$

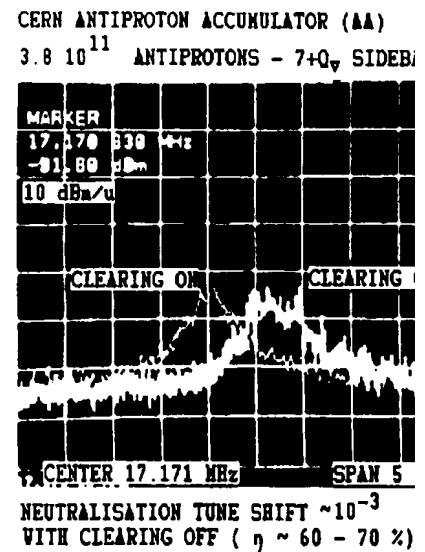


Fig. 4 AA transverse vertical Schottky scan showing the band  $(7 + Q_v)f_{mc}$  with clearing electrodes turned OFF (full neutralisation) and ON (little neutralisation)

where  $dn/dv_x dv_y dv_z$  is the number of particles per unit volume around velocities  $v_x v_y v_z$  and  $k = 1.4 \times 10^{-23}$  J/K or  $k = 8.6 \times 10^{-5}$  eV/K is Boltzmann's constant.

One finds successive mean velocities by integration

$$\langle v \rangle = v_m = \left( \frac{8kT}{\pi m} \right)^{\frac{1}{2}}$$

$$\langle v^2 \rangle^{\frac{1}{2}} = v_{rms} = \left( \frac{3kT}{m} \right)^{\frac{1}{2}}$$

$$\langle |v_x| \rangle = \langle |v_y| \rangle = \langle |v_z| \rangle = v_{||} = \left( \frac{2kT}{\pi m} \right)^{\frac{1}{2}}$$

with naturally

$$\langle v_x \rangle = \langle v_y \rangle = \langle v_z \rangle = 0,$$

so that

$$v_m = 2v_{||} \quad (\text{Annex I}).$$

The mean kinetic energy is

$$E = \left\langle \frac{1}{2} m v^2 \right\rangle = \frac{1}{2} m v_{rms}^2 = \frac{3}{2} kT. \quad (26)$$

Table 3 illustrates for different molecules the relation between energy, temperature, and velocity.

Table 3  
Relation  $T$ (K),  $E$ (eV),  $v_{||}$  ( $\text{ms}^{-1}$ ) for various molecules

|                   | A     | $v_{rms}$         | $v_{  }$          |
|-------------------|-------|-------------------|-------------------|
| H                 | 1     | $2.7 \times 10^3$ | $3.7 \times 10^3$ |
| H <sub>2</sub>    | 2     | $1.9 \times 10^3$ | $2.6 \times 10^3$ |
| H <sub>2</sub> O  | 18    | $6.5 \times 10^2$ | $1.0 \times 10^3$ |
| CO/N <sub>2</sub> | 28    | $5.2 \times 10^2$ | $0.7 \times 10^3$ |
| CO <sub>2</sub>   | 44    | $4.1 \times 10^2$ | $0.6 \times 10^2$ |
| e-                | 1/836 | $1.2 \times 10^5$ | $1.6 \times 10^5$ |

$T = 300$  K,  $E = \begin{cases} 6.3 \times 10^{-21} \text{ joule} \\ 3.9 \times 10^{-2} \text{ eV} \end{cases}$

|                |        |                   |                    |
|----------------|--------|-------------------|--------------------|
| H <sub>2</sub> | 2      | $9.8 \times 10^3$ | $13.5 \times 10^3$ |
| N <sub>2</sub> | 28     | $2.6 \times 10^3$ | $3.6 \times 10^3$  |
| e-             | 1/1836 | $6.0 \times 10^3$ | $8.3 \times 10^3$  |

$T = 7.8 \times 10^3$  K,  $E = 1$  eV

|                |        |                   |                    |
|----------------|--------|-------------------|--------------------|
| H <sub>2</sub> | 2      | $3.1 \times 10^4$ | $4.3 \times 10^4$  |
| N <sub>2</sub> | 28     | $8.3 \times 10^3$ | $11.5 \times 10^3$ |
| e-             | 1/1836 | $1.9 \times 10^6$ | $2.6 \times 10^4$  |

$T = 7.8 \times 10^3$  K,  $E = 10$  eV

### 3.2 The electric field and the potential well

Before analysing the motion of the ions, we compute the fields which act on the ions and the electrons which have been 'just created'. In the absence of external fields, an electric field is produced by the circulating beam. This field defines a potential and the value of the potential is fixed by the fact that the vacuum chamber is at ground potential. We consider the simplified case of a circular beam in a



circular chamber, where  $a$  is the radius of the beam with uniform density in real space,  $r$  the radial variable and  $r_0$  the vacuum chamber radius. We have seen already that the field can be calculated with Gauss's law:

$$\varepsilon_r = \begin{cases} \frac{e\lambda}{2\pi\varepsilon_0} \frac{r}{a^2} & r \leq a \\ \frac{e\lambda}{2\pi\varepsilon_0} \frac{1}{r} & r \geq a \end{cases} \quad (27)$$

The potential is obtained by integration

$$V = - \int \varepsilon_r dr, \quad (28)$$

the constant being fixed by the condition

$$V = 0 \quad \text{for } r = r_0.$$

The result is

$$V = \frac{e\lambda}{2\pi\varepsilon_0} \begin{cases} -\frac{r^2}{2a^2} + \frac{1}{2} + \ln\left(\frac{r_0}{a}\right) & r \leq a \\ \ln\frac{r_0}{r} & r \geq a. \end{cases} \quad (29)$$

Figure 5 represents the potential for our nominal beam with 100 mA circulating current, a vacuum chamber of 100 mm diameter and different beam sizes. Figure 6 represents the value of the central potential  $V_0$  for different ratios  $a/r_0$ . The motion of ions or electrons in this field is very simple: in a beam of electrons or antiprotons, electrons are chased and arrive on the wall with an energy  $eV_0$ , ions are trapped if their transverse energy is less than  $eV_0$ . Since the probability of energy transfer larger than the potential well is very small, ions are always trapped (see section 5 for stability considerations in bunched beams).

The detailed calculation of the potential well for non-cylindrical geometries has been made [26]. We give in Annex II the resulting formulae to be used in practical calculations.

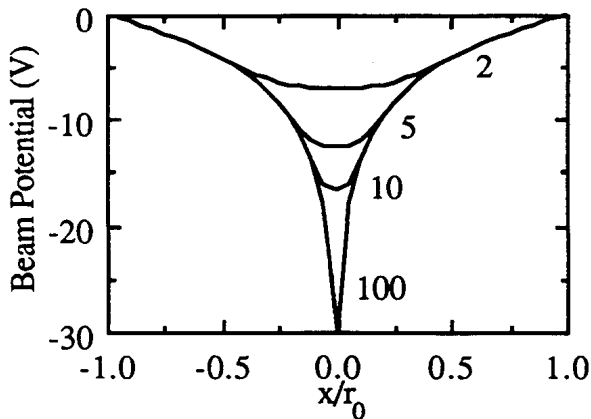


Fig. 5 Potential well for different beam sizes. The number attached to each curve is the ratio  $r_0/a$ .

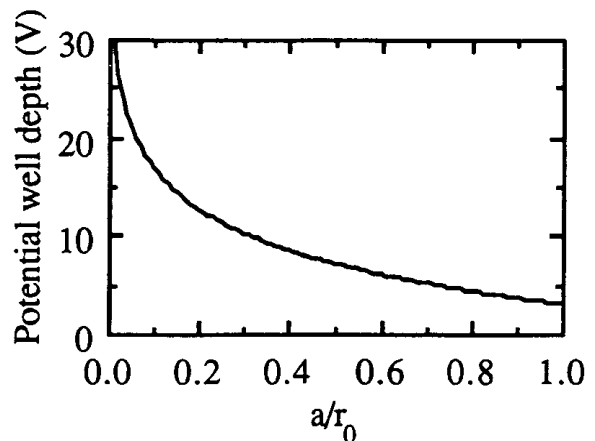


Fig. 6 Depth of the potential well vs beam size.

Figure 7 is the result of the calculation made for the ISR. These potential wells have been directly measured [34, 83].

If the vacuum chamber size or the beam size varies in a long straight section, the electron or ion will drift towards the deepest potential well. The kinetic energy gained in the process can be considerably higher than the thermal energies since the variations of potential energy can reach several tens of eV, while the thermal energies are of the order of  $10^{-2}$  eV

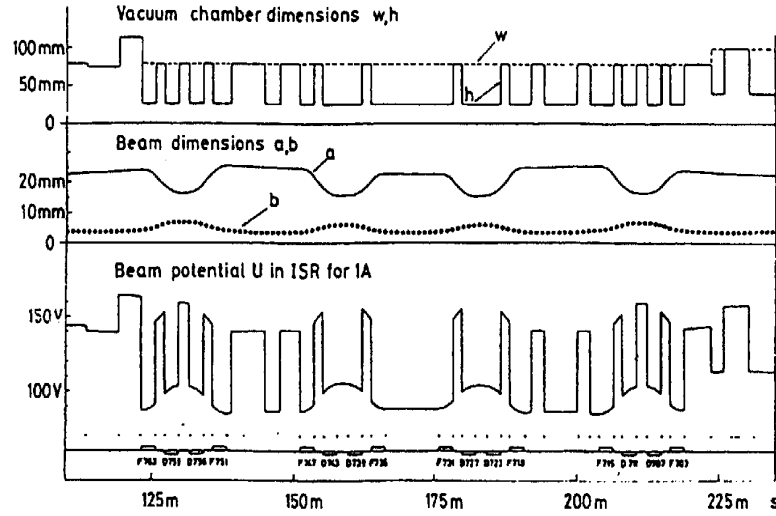


Fig. 7 Beam potential in the ISR. The position of the clearing electrodes is indicated by dots.

### 3.3 The effect of the magnetic field

The motion of a particle in a uniform magnetic field is simple, it is the well known cyclotron motion [3]. When the field has a gradient perpendicular to the field direction, a drift of the particle occurs which is called the gradient drift. If the gradient of the field is in the direction of the field there is a containment effect called the magnetic mirror. This section studies these three effects.

#### 3.3.1 Cyclotron motion

In all these problems one separates the velocity into its two components (Fig. 8):

$\vec{v}_{\parallel}$  parallel to  $\vec{B}$

$\vec{v}_{\perp}$  perpendicular to  $\vec{B}$

with

$$v^2 = v_{\parallel}^2 + v_{\perp}^2. \quad (30)$$

If the field is uniform  $\partial B / \partial r = 0$ , then the velocity along the field  $\vec{v}_{\parallel}$  is uniform and unchanged. The perpendicular velocity  $v_{\perp}$  induces a force and therefore an acceleration.

$$m \frac{d\vec{v}_{\perp}}{dt} = e \vec{v}_{\perp} \times \vec{B}.$$

This is a central force perpendicular to  $\vec{v}_{\perp}$  which gives a circular motion, the radius  $r$  of the circle is obtained by equating the central acceleration to the centrifugal force.

$$e v_{\perp} \cdot B = m \frac{v_{\perp}^2}{r}$$

The angular frequency  $\omega_c$  also called cyclotron angular frequency is

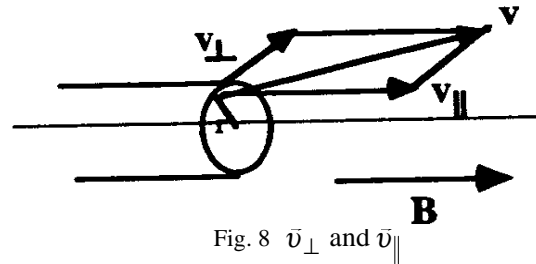


Fig. 8  $\vec{v}_{\perp}$  and  $\vec{v}_{\parallel}$

$$\omega_c = \frac{v_{\perp}}{r} \quad (31)$$

which gives

$$r = \frac{m v_{\perp}}{e B} \quad (32)$$

$$\omega_c = \frac{e B}{m} \quad (33)$$

with the remarkable result that  $\omega_c$  does not depend on  $v_{\perp}$  : in a given field the larger the velocity the longer the radius, but the frequency does not change for a given particle.

### 3.3.2 Effect of a transverse gradient (the gradient drift)

We have already seen that

$$v_{\perp} = r \frac{e B}{m} = r \omega_c$$

but here

$$B = B_0 + \frac{\partial B}{\partial x} \cdot x.$$

The projection of the velocity on z gives (with  $x = r \cos \omega_c t$ )

$$v_z = v_{\perp} \cos \omega_c t = r \frac{e B_0}{m} \cos \omega_c t + r \frac{e}{m} \frac{\partial B}{\partial x} \cdot r \cos \omega_c t \cdot \cos \omega_c t .$$

Then the mean velocity is not zero, corresponding to a drift

$$v_D = \langle v_z \rangle = \frac{1}{2} r^2 \frac{e B}{m} \frac{1}{B} \frac{\partial B}{\partial x} .$$

This calculation only applies if the field variation over the cyclotron motion is small, that is, if

$$r \frac{\partial B}{\partial x} \ll B .$$

This effect is called the gradient drift. It can also be written:

$$v_D = \frac{1}{2 \omega_c} v_{\perp}^2 \frac{1}{B} \frac{dB}{dx} .$$

This gradient can only be created by a curvature of the magnetic field, particles with a velocity parallel to the magnetic field  $v_{\parallel}$  will have to follow the field lines. This curvature will give an additional drift [3] so that the final drift can be written as

$$v_D = \frac{1}{\omega_c} \left( v_{\parallel}^2 + \frac{1}{2} v_{\perp}^2 \right) \frac{1}{B} \frac{\partial B}{\partial x} . \quad (34)$$

### 3.3.3 Effect of a longitudinal gradient (the magnetic mirror)

We assume that  $B_z$  changes with  $z$ . This gives a set of lines of force as sketched in Fig. 9. When the particles move to the right towards higher fields, the field lines are more dense. A variation of flux through the orbit would induce an electromotive force and therefore an exchange of energy between the static magnetic field and the particles. This is not possible so the flux circled by the particle is constant. (The exact demonstration makes use of the action integral (see Jackson [3] p. 422).)

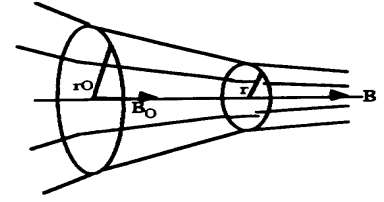


Fig. 9 The magnetic mirror.

$$\pi B r^2 = \pi B_0 r_0^2$$

or, using the equation of the cyclotronic motion,

$$v_{\perp}^2 = v_{\perp 0}^2 \frac{B}{B_0}.$$

Since the kinetic energy of the particle is conserved  $v^2 = v_0^2$ . So that (Eq. 29)

$$v_{\parallel}^2 = v_0^2 - v_{\perp 0}^2 \frac{B}{B_0}. \quad (35)$$

If  $B$  becomes large enough, then  $v_{\parallel}^2 = 0$ . The motion of the particle is stopped. Detailed calculations show that the particle in fact spirals back.

Looking at Eq. 34 it is clear that the particle will be trapped if  $v_{\parallel}$  can reach zero, that is if

$$\left| \frac{v_{\parallel 0}}{v_{\perp 0}} \right| < \left( \frac{B}{B_0} - 1 \right)^{\frac{1}{2}}. \quad (36)$$

With an isotropic distribution of speed at the time of creation of the particle the proportion of particles trapped will be:

$$R = 1 - \frac{B_0}{B}. \quad (37)$$

### 3.4 Combined effects of $\epsilon$ and $B$ (the cross-field drift)

We consider the magnetic field of a magnet and the electric field of the beam (Fig. 10). The electric force is:  $e\vec{\epsilon}$  and the magnetic force is:  $e \cdot \vec{v}_{\perp} \times \vec{B}$ . At equilibrium

$$v_{\perp} = \frac{\epsilon}{B}.$$

This equilibrium can only be attained if  $v_{\perp}$  can reach  $\epsilon/B$  that is if

$$\frac{\epsilon}{B} < c.$$

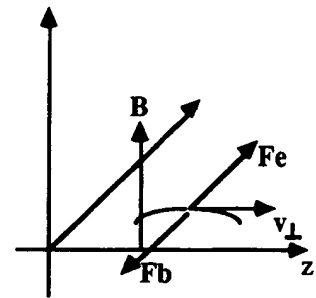


Figure 10: Cross-field drift

A rather simple analysis shows that indeed if  $\epsilon < cB$  this equilibrium is always reached. However, the time it takes to reach that equilibrium is approximately the time it takes for the field to accelerate the particle to an energy corresponding to the velocity  $v_{\perp}$ . In practice during the acceleration phase where  $v_{\perp} \cdot B$  is very small compared to  $\epsilon$ , the magnetic field can be neglected. If  $\epsilon/B > c$  this equilibrium will never be reached, the magnetic field can be neglected. In all the practical cases which will be considered, the motion can be described by a pure acceleration or a pure drift. Where  $\epsilon/B \ll c$  the equilibrium is reached in less than a  $\mu s$  and the transverse displacement is less than a  $\mu m$ .